Chapter 8 solution

8.2 (a) $B(\hat{\theta}) = 0$

(b) $E(\hat{\theta}) = \theta + 5$

8.4 (a) $MSE(\hat{\theta}) = V(\hat{\theta})$

(b) " $>$ "

8.8 (a)

$E(\hat{\theta}_1) = \theta$ (unbiased)

$E(\hat{\theta}_2) = \frac{2\theta}{2} = \theta$ (")

$E(\hat{\theta}_3) = \theta$ (unbiased)

$E(\hat{\theta}_4) = \frac{\theta}{3}$ (biased)

$E(\hat{\theta}_5) = \theta$ (unbiased)

(b) $Var(\hat{\theta}_1) = \theta^2$

$Var(\hat{\theta}_2) = \frac{1}{4}(\theta^2 + \theta^2) = \frac{\theta^2}{2}$

$Var(\hat{\theta}_3) = \frac{1}{9}(\theta^2 + 4\theta^2) = \frac{5}{9}\theta^2$

$Var(\hat{\theta}_5) = \frac{\theta^2}{3}$

Hence $\hat{\theta}_5$ has the smallest variance.

8.14

(a) $f(y) = \alpha y^{\alpha-1}/\theta^{\alpha}, \quad 0 \le y \le \theta$.

$$F(y) = \int_0^y \alpha y^{\alpha-1}/\theta^{\alpha} \, dy$$

$$= y^{\alpha}/\theta^{\alpha} \Big|_0^y = \frac{y^{\alpha}}{\theta^{\alpha}}, \quad 0 \le y \le \theta$$

$$\Rightarrow f_{Y_{(n)}}(y) = n \cdot \frac{\alpha y^{\alpha-1}}{\theta^{\alpha}} \cdot \left(\frac{y^{\alpha}}{\theta^{\alpha}}\right)^{n-1}$$

$$= \frac{n\alpha \, y^{n\alpha-1}}{\theta^{n\alpha}}, \quad 0 \le y \le \theta$$

$$\Rightarrow E(\hat{\theta}) = E(Y_{(n)}) = \int_0^{\theta} \frac{n\alpha y^{n\alpha}}{\theta^{n\alpha}} \, dy$$

$$= \frac{n\alpha}{n\alpha+1} \cdot \frac{1}{\theta^{n\alpha}} \cdot y^{n\alpha+1} \Big|_0^{\theta}$$

$$= \frac{n\alpha}{n\alpha+1} \cdot \theta \ne \theta$$

Hence $\hat{\theta}$ is biased.

(b) $\frac{n\alpha+1}{n\alpha} \hat{\theta}$. is unbiased for θ.

(c) $$\mathrm{var}(\hat{\theta}) = E(\hat{\theta}^2) \cancel{\theta} - [E(\hat{\theta})]^2$$

$$= \frac{n\alpha}{n\alpha+2} \theta^2 - \left(\frac{n\alpha}{n\alpha+1}\right)^2 \theta^2$$

$$= \frac{n\alpha}{(n\alpha+2)(n\alpha+1)^2} \theta^2$$

Hence, $MSE(\hat{\theta}) = \frac{n\alpha}{(n\alpha+2)(n\alpha+1)^2}\theta^2 + \left(\frac{1}{n\alpha+1}\right)^2\theta^2$

$$= \frac{2}{(n\alpha+2)(n\alpha+1)}\theta^2$$

8.56 (a) $\hat{p} = 0.45$

98% CI :

$$\hat{p} \pm z_{0.01} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.45 \pm 2.33 \cdot \sqrt{\frac{0.45 \times 0.55}{800}}$$

$$= (0.41, 0.49)$$

(b) No. No.

8.58 $\bar{Y} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$

$$= 5.4 \pm 1.96 \times \frac{3.1}{\sqrt{500}}$$

$$= (5.13, 5.67)$$

8.64 (a) $\hat{p}_1 = .35$, $\hat{p}_2 = .77$

$$\hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$= .35 - .77 \pm 2.33 \cdot \sqrt{\frac{.35 \times 0.65}{340} + \frac{.77 \times .23}{150}}$$

$$= (-0.52, -0.32)$$

(b) No. Because the difference between these two age groups does not exceed as large as 0.5?.

8.96 $s^2 = 63.51$

$$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}} , \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}} \right)$$

$$= \left(\frac{9 \times 63.51}{16.92} , \frac{9 \times 63.51}{3.33} \right)$$

$$= (33.78 , 171.65)$$